$$\Sigma mxy = \rho\sin^2\beta\int_{-\frac{1}{2}a}^{\frac{1}{2}a}\int_{-\frac{1}{2}b}^{\frac{1}{2}b}(x+y\cos\beta)ydxdy = \tfrac{1}{24}mb^2\sin2\beta.$$

$$\therefore\ \tan2\theta = \frac{b^2\sin2\beta}{a^2+b^2(\cos^2\beta-\sin^2\beta)} = \frac{b^2\sin2\beta}{a^2+b^2\cos2\beta}.$$

Let A, B be the principal moments.

$\therefore\ A\cos^2\theta+B\sin^2\theta=\frac{1}{12}m(a^2+b^2\cos^2\beta)\ldots\ldots(1)$.

$A\sin^2\theta+B\cos^2\theta=\frac{1}{12}mb^2\sin^2\beta\ldots\ldots(2)$.

(1)+(2) gives $A+B=\frac{1}{12}m(a^2+b^2)$.

(1)−(2) gives $A-B=\frac{1}{12}m(a^2+b^2\cos2\beta)\sec2\theta$

$$=\tfrac{1}{12}m\sqrt{(a^4+b^4+2a^2b^2\cos2\beta)}.$$

$\therefore\ A=\frac{1}{24}m[a^2+b^2+\sqrt{(a^4+b^4+2a^2b^2\cos2\beta)}]$.

$B=\frac{1}{24}m[a^2+b^2-\sqrt{(a^4+b^4+2a^2b^2\cos2\beta)}]$.

DIOPHANTINE ANALYSIS.

76. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

It is required to find four positive numbers such that if each be diminished by twice the cube of their sum the four remainders will be rational cubes.

Solution by the PROPOSER.

Let u, v, x, y be the numbers.

Then $u-2(u+v+x+y)^3=a^3/h^3(x+y+u+v)^3$, suppose.

$v-2(u+v+x+y)^3=b^3/h^3(x+y+u+v)^3$, suppose.

$x-2(u+v+x+y)^3=c^3/h^3(x+y+u+v)^3$, suppose.

$y-2(u+v+x+y)^3=d^3/h^3(x+y+u+v)^3$, suppose.

Adding we get

$$u+v+x+y-8(u+v+x+y)^3=\frac{a^3+b^3+c^3+d^3}{h^3}(u+v+x+y)^3.$$

Let $a^3+b^3+c^3+d^3=h^3$.

$\therefore\ u+v+x+y=9(u+v+x+y)^3$. $\therefore\ u+v+x+y=\frac{1}{3}$.

$$\therefore\ u=\frac{a^3+2h^3}{27h^3},\ v=\frac{b^3+2h^3}{27h^3},\ x=\frac{c^3+2h^3}{27h^3},\ y=\frac{d^3+2h^3}{27h^3}.$$

Let $a=1$, $b=5$, $c=7$, $d=12$, $h=13$.

$\therefore\ u=\frac{4395}{59319},\ v=\frac{4519}{59319},\ x=\frac{4737}{59319},\ y=\frac{6122}{59319}$.

Let $a=4$, $b=7$, $c=8$, $d=17$, $h=18$.

$\therefore\ u=\frac{11728}{157464},\ v=\frac{12007}{157464},\ x=\frac{12176}{157464},\ y=\frac{16577}{157464}$.

Other values can be found for u, v, x, y.

77. **Proposed by JOHN M. COLAW, A. M., Monterey, Va.**

Find (1) three consecutive numbers whose sum is a cube, and (2) three consecutive numbers the sum of whose cubes is a cube.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

(1). Let $n-1$, n, and $n+1$ be any three consecutive numbers.

Then $(n-1)+n+(n+1)=3n=$a cube$=27m^3$.

Whence $n=9m^3$.

$\therefore$ $9m^3-1$, $9m^3$, and $9m^3+1$ are the general expressions for three consecutive numbers whose sum is a cube.

Take $m=1$; then $8+9+10=27=3^3$.

Take $m=2$; then $71+72+73=216=6^3$; etc.

(2). $(n-1)^3+n^3+(n+1)^3=3n^3+6n=$a cube$=27m^3$.

Whence $n^3+2n=9m^3$.

Put $m=an$; then $n^3+2n=9a^3n^3$.

Whence $n^2+2=9a^3n^2$; and $n^2=2/(9a^3-1)$.

To obtain n integral, a must be fractional.

Put $a=1/b$; then $n^2=2b^3/(9-b^3)$.

To avoid imaginary results, $b<2\frac{1}{12}$.

The only integral values that can be assigned to b are 1 and 2.

Take $b=1$; then $n=\frac{1}{2}$.

Whence $(-\frac{1}{2})^3+(\frac{1}{2})^3+(\frac{3}{2})^3=(\frac{3}{2})^3$.

Take $b=2$; then $n=4$.

Whence $3^3+4^3+5^3=6^3$.

This is the only set of three consecutive integers the sum of whose cubes is a cube.

Fractional values of b give fractional values for n.

When $b=0$, $n=0$.

Whence $(-1)^3+0^3+1^3=0^3$.

Also solved by *CHARLES C. CROSS, JOSIAH H. DRUMMOND, ALOIS F. KOVARIK, NELSON L. RORAY, J. SCHEFFER, ELMER SCHUYLER*, and *G. B. M. ZERR*.

AVERAGE AND PROBABILITY.

81. **Proposed by LON C. WALKER, Assistant in Mathematics, Leland Stanford, Jr., University, Palo Alto, Cal.**

Find (1) the mean distance of all points on a side of an equilateral triangle from the opposite vertex; and (2), the average length of a line drawn at random across an equilateral triangle.